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A check determination of the velocity of light

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With 6 figures in the text

Summary of a preceding determination

In Arkiv för fysik, Bd 2, Nr 15, 1950, called 49 below, the author in detail relates a determination of the velocity of light, carried out in 1949. The error limits of the result were thought to be too narrow. For that reason a check determination under altered conditions was valuable. First it may be appropriate to give a brief summary of the preceding work.

The method is based on Fizeau's principle and the apparatus is of the Kerr cell-phototube type. It is portable and accordingly useful for measurements of distances in the field. In this case the velocity is supposed to be known. By kind permission of Professor Siegbahn the fundamental experiments were carried out in the Nobel Institute of Physics, where the original apparatus was put into operation in 1947. It was tested at Lovö and the result was good. After that an improved model, the geodimeter, was built by Aga and used in the present determinations.

Figs. 1 and 2 will facilitate a description of the geodimeter. In Fig. 1 L is the source of light. The intensity of the light is varied with the frequency 8 Mc/sec by the Kerr cell Kc. The mirror M_1 focusses the light into a beam directed towards the distant plane mirror M_3. By the receiving optics the returning light is thrown on the cathode of the multiplying phototube Ph.

In Fig. 2 is shown the electric circuits and in Fig. 3 the curve of the light intensity as dependent on the voltage of the Kerr cell. This voltage consists of a high frequency part, 8 Mc, 2000 volts, and a 50-cycle part, 5000 volts in amplitude. The high frequency emanates from the crystal-controlled transmitter Cr–HA. Tr delivers the 50-cycle voltage, which is given a rectangular wave shape by means of resistances and neon tubes. On account of the 50-cycle voltage the high frequency light variations *change their phase* 100 times a second. The anode of the phototube Ph is supplied by high frequency voltage from Cr–HA also. By the shifter SwI the phase of this voltage can be manually reversed. No 50-cycle voltage is supplied to Ph. Due to the photo-currents there will arise 50-cycle voltage pulses in the anode resistance *70*. The individual magnitude of every pulse depends on the actual phase difference between the high frequency feeding of the anode and the entering light variations. The phase difference, in turn, directly depends on the distance to the mirror M_3. The instrument I receives the 50-cycle pulses alternately from the valve *1* and *2*. The valves are alternately blocked every second half-cycle by the 50-cycle volt-

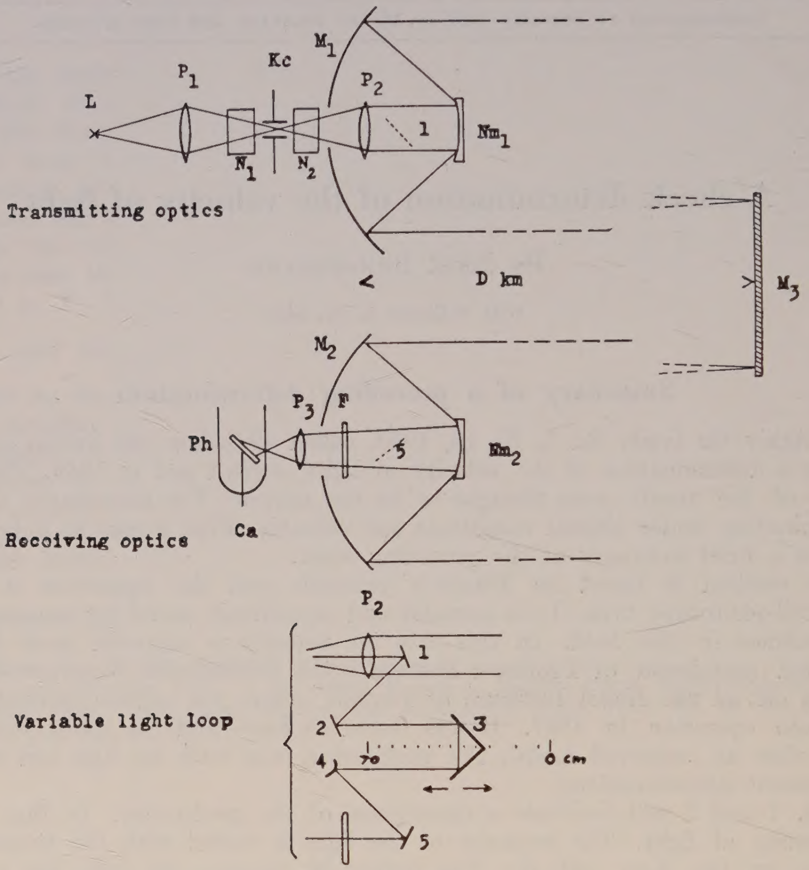

Fig. 1.

age from *Tr* on the grids *a* and *b*. By the shifter *SwII* the phase of this 50-cycle control voltage can be manually reversed.

The slowly reacting direct current instrument *I* shows the difference

$$i = i_1 - i_2 \tag{1}$$

of the mean currents i_1 and i_2 delivered respectively by the valves 1 and 2. The current i equals zero every 9th meter (zero-points) of increased distance to the mirror M_3.

The intensity of the light falling on the photo-cathode must contain a series of components as:

$$\sum_{p=1}^{p=\infty} A_p \cdot \sin p\,\omega\,t + Q, \quad \sum_{p=1}^{p=\infty} A_p \cdot \sin p\,(\omega\,t + \pi) + Q. \tag{2}$$

A_p and Q = constants, $\omega = 2\,\pi\,n$, n = frequency, t = time, p = integer.

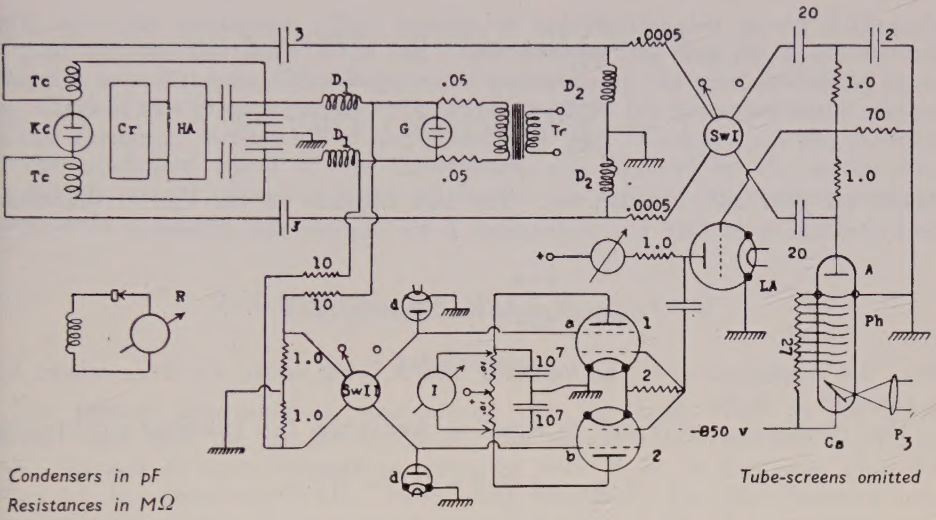

Fig. 2.

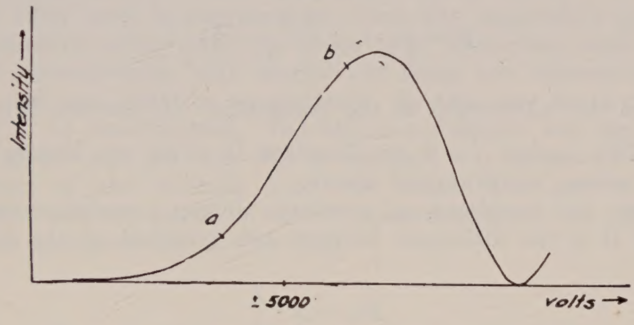

Fig. 3.

The two forms of the eq. (2) depends on whether the light is emitted during a positive or a negative 50-cycle half-cycle. π denotes the high frequency phase shift mentioned above (100 times a second).

The photo-anode is supplied by the same high frequency voltage. Accordingly the sensitivity of the tube contains an analogous series of the following shape:

$$\sum_{p=1}^{p=\infty} B_p \cdot \sin p\,\omega\,(t - t_1 - 2\,D/c) + R. \tag{3}$$

Here we have no 50-cycle and no difference of phase π. The sensitivity never being negative R is positive. t_1 mainly depends on electrical delay within the apparatus and $2\,D/c$ is the time which the light uses on the long way to the distant mirror and back again with the velocity c. The photo-current developed in the tube is a product of the intensity of the received light and of the sensitivity of the tube, that is a product of two sines. Therefore it must contain a series of terms as:

$$i_+ = +\sum_{p=1}^{p=\infty} H_p \cdot \cos p\,\omega\,(t_1 - 2\,D/c) + S,$$
$$i_- = -\sum_{p=1}^{p=\infty} H_p \cdot \cos p\,\omega\,(t_1 - 2\,D/c) + S \tag{4}$$

where all $p\,\omega\,t$ are cancelled out. The sign depends on the sign of the actual 50-cycle half-cycle. By the instrument I we observe the difference

$$i_a = i_+ - i_- = \sum_{p=1}^{p=\infty} 2\,H_p \cdot \cos p\,\omega\,(t_1 - 2\,D/c). \tag{5}$$

For, the amplification of the valves *1* and *2* being equal, $i = k \cdot i_a$, where k is a constant (i in eq. 1).

The current i_a (or i) will be equal to zero when the following eq. prevails:

$$\omega \cdot (t_1 - 2\,D/c) = (1/2 - N) \cdot \pi, \tag{6}$$

where N is an arbitrary integer. Eq. (6) gives

$$D = \left(n \cdot t_1 + \frac{2\,N - 1}{4}\right) \cdot \frac{\lambda}{2}, \tag{7}$$

where $\lambda = \frac{c}{n}$ is the wavelength of the transmitter. In the case of $n = 8.332$ Mc λ is 36.0 m. This makes $i = 0$ at distances D which are integral multiples of the length 9 meters as mentioned above.

n and t_1 can be considered as constants during a certain time. Then they will vanish if D is the difference between two positions of the mirror. In this case we get

$$D_N = N \cdot \frac{\lambda}{4} \tag{8}$$

D_N being known we get λ. N is easy to determine from approximate values. The frequency corresponding to λ being n and known, we get the velocity in air from

$$c_a = n \cdot \lambda. \tag{9}$$

To get the velocity in a vacuum we have to multiply c_a by the refractive index of air, corrected for group velocity.

In the case of practical measurements, the mirror M_3 is placed close to a zero-point. The frequency of the transmitter, being adjustable within certain limits, is changed until the zero-point and the mirror coincide. The actual frequency is read off. One observation by the geodimeter consists of four readings corresponding to the four possible position-combinations of the two phase shifters *SwI* and *SwII*. Hereby electrical asymmetry is cancelled out.[1]

In eq. (8) we presuppose two positions of the mirror. The one is that of the mirror M_3, and the second we get by a continually variable light path, built into the geodimeter. The position of the right-angled mirror 3 (Fig. 1) is read off on the scale 0–70 cm. The distance D_N was the 7 km Survey base line near Enköping. The frequency n increasing by the crystal working time, was checked two times a day by radio transmission to the frequency check station of the Telegraph Service. The dependence of the frequency on the position of the condenser wheel of the transmitter, was checked by the service too. To get the actual refractive index of the air, temperature and pressure readings were made.

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[1] In 49, p. 123 line 4 and 6 from foot: Instead of phase, read: intensity of light.

The present determination

Introduction

The present determination was carried out in August 1950 on the 6 km normal base line on the isle of Öland. The measurements and the apparatus were changed in some respects as compared with 1949. It happened that the index of the frequency wheel was slightly displaced. For that reason a new curve was drawn from new checked frequency values. The change of the index could merely result in a parallel displacement of the curve.

The yellow filter used in the previous work was removed; a new determination of the effective colour with the green filter alone was made.

The actual measurement was carried out using two mirrors in the field at distances of 600 and 6000 m respectively. In this case the built in variable light loop (Fig. 1) was omitted. The effective distance was that between the two mirrors (5413 m). As the 600 m distance was accurately known too, a new calabration of the variable loop was made. This gave a value on the distance to the first zero-point. A direct measurement of the light path in the loop was made (compare 49, p. 133–134).

Frequency curve, colour and refractive index

From table I we get the connection between frequency and adjustment of the frequency wheel C (compare 49, p. 127). The shape of the resulting curve agrees very well with the old one (49, Fig. 9), the origin however is changed. The new curve is shown in Fig. 4.

Normal frequency is 8332387 Hz at $C = 4.0$. Using the curve one has to take into consideration that the cm-distances in the right margin, proportionate to a change in the frequency, are valid for the distance 6907 m (Enköping) and here has to be reduced by a factor $\frac{5413}{6907}$ (49, foot of p. 136).

The green colour of the light used was determined in the same way as for the green-yellow filter (49, p. 126). The voltage on the lamp was 4 volts and the curve which was obtained is shown in Fig. 5. Taken from the curve, the effective colour is 5050 Å. The blue absorption of the air and the voltage often being 3.5 volts justifies an increase to 5150 Å. By eq. (18), (19) in 49 we get the refractive index

$$\mu_g = 1.0003064.$$

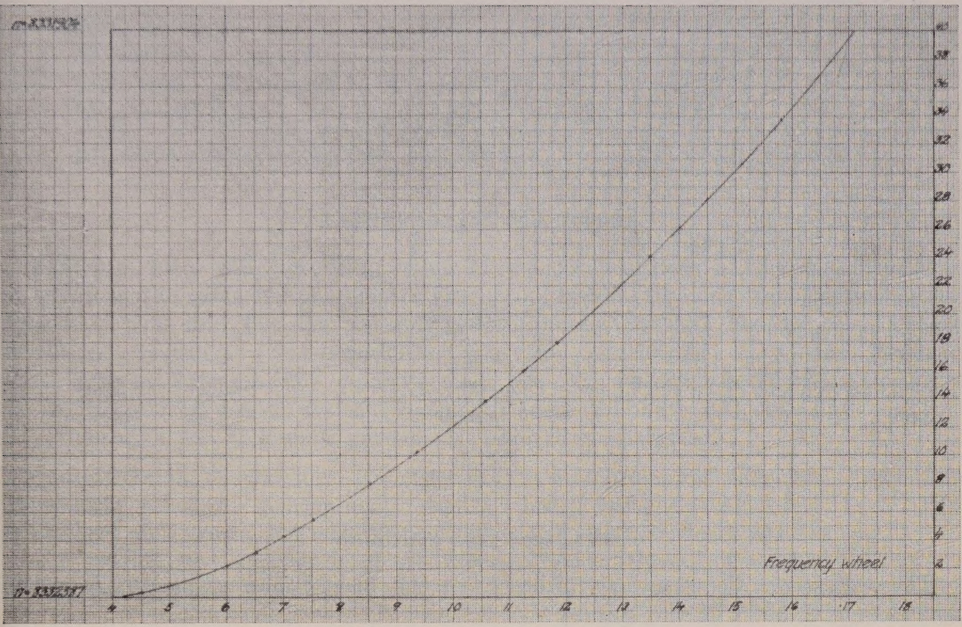
Table I

Checking of the frequency. Time = working time of the crystal (hours).

Date in 1950	Hour	Crystal wheel	Frequency	Time
28.8	21.45	10	8332228	4
29.8	21.50	10	253	8
30.8	21.45	10	250	13
31.8	20.50	10	265	16
31.8	21.48	10	265	17
1.9	21.50	10	258	21
2.9	20.47	14	110	24
2.9	21.47	14	113	25
4.9	20.45	14	105	29
11.9	20.53	14	150	54
12.9	20.31	14	141	59
30.9	20.47	10	316	66
30.9	21.49	10	334	67
11.10	12.30	14	145	80
	32	4	465	
	34	14	149	
	36	9	347	
	38	14	149	
	40	19	8331851	
	42	14	8332126	
	44	16.5	002	
	46	14	134	
	48	11.5	252	
	50	14	148	
	52	6.5	429	
	54	14	149	
	56	24	8331425	
	58	29	8330768	
	13.00	34	8329638	
	02	29	8330738	
	04	24	8331382	
	06	14	8332107	

Arrangements in the field

The geodimeter was placed 5 m above the ground and covered by a masonite shed. The convex 600-m mirror was situated on a fixed stand, 1.5 m in height. The plane 6-km mirror, finally, was placed on a stand, 6 m in height and covered by roof and three walls of tent-cloth. The geodimeter and the mirrors were in the same vertical plane. The shift of the measurements from one mirror to the other was carried out by a slight change in the inclination of the geodimeter. The focusing of the optics was kept unaltered. The arrangement of the geodimeter and the mirrors is shown in Fig 6. *N.B.* is the northern ground knob of the base line. There is a spare marking 24 m in the extension of the base. *S.B.* is the southern ground knob. *G* is the geodimeter and M_1 the 600-m mirror, close to a knob, $N.B._{600}$. For the rest, there are knobs at every 600 m the whole base through. M_2 is the plane 6-km mirror.


40
38
36
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Frequency wheel
4
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17
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Fig. 4.

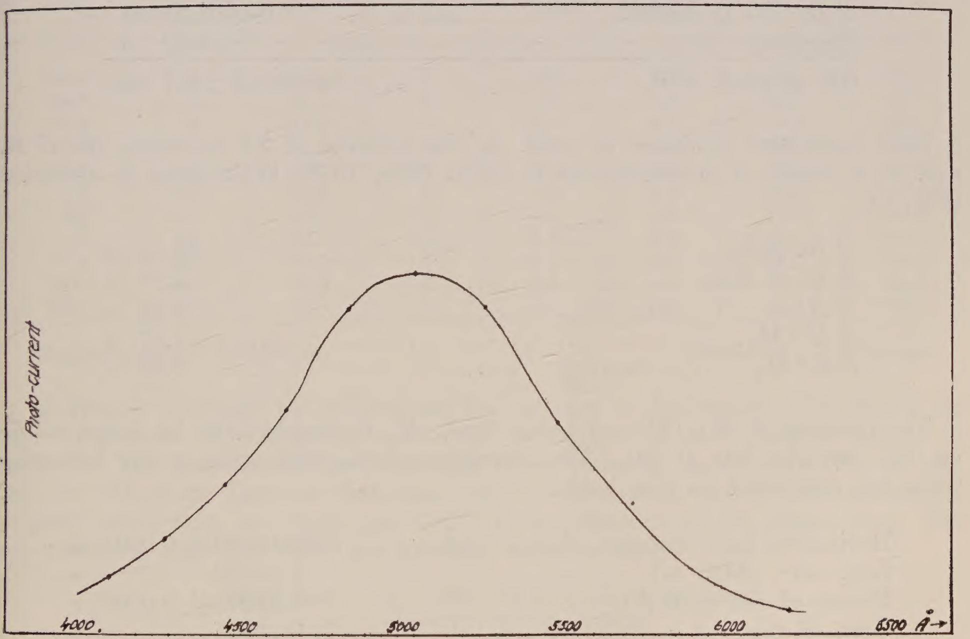
Photo-current
4000
4500
5000
5500
6000
6500 Å →

Fig. 5.

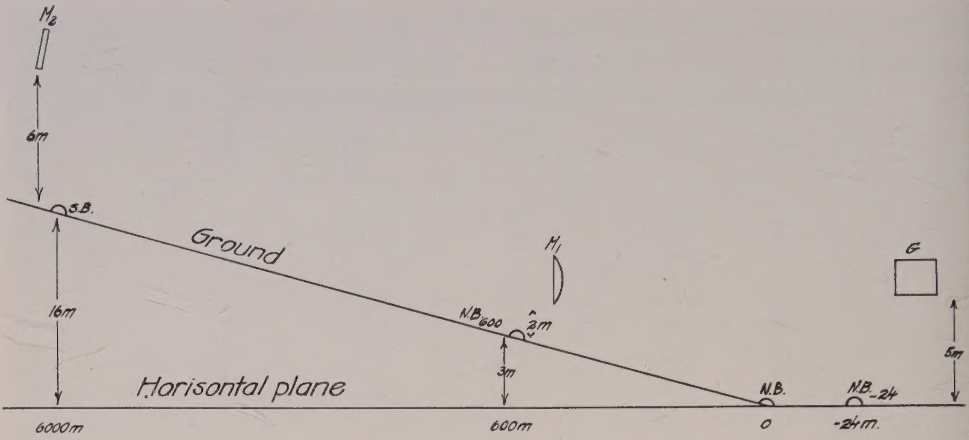

Fig. 6.

Distances and altitudes

The values on distances and altitudes of the base knobs are taken from the manuscript G 3593 of the Survey:

N.B.–S.B.	+ 6000004.39 mm
Corr. for pole-altitude	+ 1.20 »
N.B.–600 m knob	— 600003.21 »
Pole corr.	— 0.12 »
600 m knob–*S.B.*	5400002.3 ± 0.7 mm

This horizontal distance is valid in the altitude of 9.8 m above the *N.B.* and is a mean of measurements in 1921, 1929, 1933. Differences in altitudes (Fig. 6):

N.B.–S.B.	16.62 m
$N.B.–N.B._{600}$	3.27 »
N.B.–G	5.55 »
$N.B.–M_1$	1.87 »
$S.B.–M_2$	6.65 »

The distance $G–M_1$ (621 m) being fixed, M_1 is imagined to be raised to lie on the straight line $G–M_2$. The corrections from inclination in the following table are computed on this basis:

Horizontal base distance $N.B._{600}–S.B.$	+ 5400.0023 ± 0.0007 m
Incl. corr. (5413 m)	+ 0.0234 »
Plumb of M_2 to S.B.	+ 2.9972 ± 0.0003 »
Corr. of tape	+ 0.0005 »
M_2-surface to plumb	—0.0225 »

Plumb of M_1 to $N.B._{600}$	$+10.1990 \pm 0.0005$	m
Corr. of tape	$+0.0015$	»
M_1-surface to plumb	$+0.1782$	»
Incl. corr. (M_1 raised to line G–M_2)	$+0.0026$	»
Line G–M_2 4.6 m above the computed base dist.	$+0.0039$	»
Effective distance between the mirrors	5413.3861 ± 0.0010	m

Calculation of the velocity

Starting with the figures in table II we get a survey of the measurements as given in the table further below. The actual measuring conditions being changed into 0° C, 760 mm Hg dry air and the frequency 8332387 Hz, we could obtain unaltered readings (zero-deflections) on the instrument I, if the distance 5413.3861 m between the mirrors M_1, M_2 was changed by certain amounts. Δ in the survey below are the means of these amounts for each day.

Δ contains the following corrections entered afterwards:

Barometerindex (— 0.8 mm Hg according to Meteorological Service)	—1.7 mm
Barometer (close to geodimeter) raised to beam level (8.9 m)	—1.7 »
Influence of voltage and colour change in the M_1-measurements (to get sufficient intensity from the convex mirror the lamp voltage was increased to 5 volts)	—1.0 »

Survey of the measurements:

Date	Frequency 8332387 +	Δ cm	Number of obs.	Weight	Velocity 299790 +	C°	mm Hg
26.8	0	—16.4	2	0.6	3.2	+13.6	760.5
28.8	4	21.8	2	0.5	0.3	15.1	757.7
29.8	8	13.9	4	1.5	4.7	15.8	752.7
30.8	15	13.9	3	1.1	4.6	14.1	754.0
31.8	20	18.7	6	3.0	2.0	11.8	754.8
5.9	38	19.4	5	1.6	1.6	14.5	756.4
6.9	43	12.7	3	1.5	5.3	11.5	755.4
7.9	48	17.2	6	2.1	2.8	13.5	745.5
9.9	55	16.2	6	3.0	3.3	11.5	751.5
10.9	60	13.9	5	2.5	4.6	12.2	754.1
12.9	68	19.0	5	2.6	1.8	14.3	751.0

Weighted mean: — 16.56 cm 299793.14 ± 0.42 km/sec

Δ already contains the corrections for increase in frequency. The reason for the frequency column is to show the treatment of the checked values in table I. The increase of the frequency takes place in small steps (49, p. 138, col. 3). They are however smoothed out in the column above. The increase is slightly more than one cycle per hour of running time of the transmitter. The results from 1949 and 1950 are brought together here:

1949 May: $c = 299\ 793.04 \pm 0.19$ km/sec
» Sept.: $c = 299\ 793.10 \pm 0.20$ »
1950 Aug.: $c = 299\ 793.14 \pm 0.42$ »

Table II

Readings on the crystal wheel during the measurement towards the first mirror (in italics) and the second mirror (M_2 in Fig. 6). Each reading in a group of four corresponds to one of the four possible position-combinations of the shifters SwI and SwII. The first figures each day are date and hour of start. The last figures are hours of conclusion.

26.8	1.7	24.15	23.25	*22.6*	*26.8*	*26.7*	*28.6*	*25.8*
21.00	15.0	—	—	*27.6*	*26.8*	*31.6*	*30.3*	*26.5*
9.2	8.3	31.8	5.9	*34.5*	*31.4*	*30.0*	*29.6*	*32.1*
9.6	7.7	21.00	20.20	*32.2*	*31.3*	*32.5*	*29.3*	*32.1*
15.7	9.5	12.0	12.2	17.2	9.7	13.1	13.1	13.7
16.7	13.1	14.3	14.7	18.2	15.5	16.5	16.2	15.4
0.0	9.2	14.2	11.8	8.9	12.0	9.8	11.0	14.0
0.0	11.4	14.0	15.5	8.9	16.0	15.5	14.6	14.6
36.0	*32.6*	10.8	*27.2*	*31.3*	*31.1*	11.5	*26.3*	*28.3*
36.0	*34.0*	16.0	*26.2*	*32.2*	*29.8*	17.6	*26.3*	*27.2*
11.4	*0.0*	11.0	*30.2*	*29.3*	*20.0*	12.9	*29.8*	*32.8*
13.9	*−12.4*	15.4	*30.8*	*29.4*	*23.9*	13.4	*31.1*	*32.8*
11.9	11.1	*17.3*	13.2	12.7	13.6	*27.6*	13.6	10.7
16.5	13.1	*20.0*	15.9	15.2	18.4	*26.2*	16.4	13.5
0.0	10.7	*26.5*	14.2	10.8	17.3	*29.1*	12.6	17.3
0.0	14.6	*29.0*	16.9	13.4	10.0	*28.1*	14.4	17.0
29.7	*33.3*	11.8	*26.9*	*25.0*	*28.3*	10.4	*26.2*	*25.0*
33.6	*33.1*	13.7	*27.5*	*27.7*	*30.1*	15.9	*26.8*	*28.6*
22.45	*0.0*	12.0	*28.8*	*28.7*	*29.4*	12.2	*31.5*	*28.6*
—	*0.0*	15.2	*27.8*	*30.6*	*29.4*	15.3	*33.6*	*25.3*
28.8 20.35	23.30	11.7	13.7	10.3	7.7	*28.0*	13.7	13.4
14.3	—	15.9	14.5	13.2	18.5	*33.1*	15.3	16.7
16.4	30.8	11.2	17.2	13.1	12.2	*25.2*	13.5	14.9
9.7	22.00	14.4	17.7	14.7	12.7	*25.2*	14.0	16.2
9.1	15.3	*26.4*	*27.5*	*31.8*	*33.5*	13.2	*29.9*	*29.2*
16.7	17.0	*27.6*	*29.0*	*34.4*	*33.2*	15.7	*30.4*	*30.4*
16.5	7.0	*27.9*	*28.7*	*17.1*	*24.7*	10.8	*25.3*	*28.1*
6.0	10.0	*27.9*	*29.4*	*20.1*	*24.9*	15.1	*28.3*	*29.0*
0.0	14.7	12.6	14.8	23.05	11.1	*30.9*	12.8	13.7
(*−10.0*)	15.8	15.7	16.1	—	17.1	*28.2*	15.4	17.4
(*−10.0*)	0.0	11.9	13.0	7.9	12.1	*22.6*	10.2	12.4
28.6	0.0	15.3	14.2	19.45	14.9	*22.7*	11.7	15.1
28.6	*19.0*	*27.0*	*27.2*	7.8	*31.2*	13.9	*29.3*	*31.8*
21.45	*27.6*	*27.9*	*26.9*	17.1	*33.2*	16.8	*32.5*	*31.9*
—	*32.1*	*23.1*	*29.8*	9.6	*31.6*	10.7	*27.7*	*29.7*
29.8 21.05	*30.4*	*24.3*	*29.8*	17.5	*29.8*	14.6	*27.8*	*29.1*
3.4	10.6	13.2	13.6	*26.2*	23.20	*29.0*	22.05	22.20
13.6	13.4	15.0	14.4	*28.8*	—	*31.3*	—	—
9.3	11.9	12.4	10.6	*34.5*	9.9	*23.6*	12.9	
9.8	13.5	14.4	12.4	*36.2*	20.15	*27.6*	19.50	
31.8	*30.3*	*27.3*	23.20	12.1	12.4	23.00	13.7	
32.8	*26.0*	*25.0*	—	17.9	17.9	—	16.3	
0.0	*29.1*	*28.0*	6.9	10.4	7.0	19.50	13.2	
0.0	*25.0*	*27.9*	20.00	15.4	17.8		16.4	

The above limits are taken from the deviations from the mean. The larger error 1950 depends on the shorter base, worse weather and the more dispersed values from the measurements towards the mirror M_1 (Fig. 6) as compared with those from the built in light loop 1949. To get the real limits of a final mean we have to take into consideration the following uncertainties:

Adjustments	$\pm 5 \times 10^{-7}$
Refr. index	$\pm 2 \times 10^{-7}$
Colour	$\pm 2 \times 10^{-7}$
Atmosph.	$\pm 2 \times 10^{-7}$
Base-line	$\pm 3 \times 10^{-7}$
Frequency	$\pm 2 \times 10^{-7}$

$$\pm \sqrt{[vv]} = \pm 7 \times 10^{-7}$$

On account of the new determinations of the colour, frequency, atmospheric conditions, and the use of a new base line the figures have been reduced somewhat as compared with those of 1949 (49, p. 139). Thus we get the result of both the years' measurements:

$$c = 299\ 793.1 \pm 0.2 \text{ km/sec.}$$

Determinations of lengths (and 1st zero-point)

Measuring lengths, e.g. first order triangle sides, we have no 600-m mirror and must use the built in variable light loop to get the actual position of the 1st zero-point (49, p. 124, 133). To calibrate the loop scale or, what amounts to the same thing, to get the position of the first zero-point if the loop scale reading is zero, an ordinary length measurement was carried out towards the mirror M_1 (Fig. 6). The distance as taken from the base line was 620.9491 m. This distance was reduced by $\frac{79}{4} \times 35.96824$ m, that is $\frac{N}{4} \times \lambda$ and we get the first zero-point to be 0.497 ± 0.010 m in front of the geodimeter-index. The same distance measured directly from the dimensions of the light loop was 0.496 ± 0.005 m. This dimension can deviate a maximum of 1 cm in different positions of the geodimeter. The value as measured in May 1949 was 0.491 and in September 0.480. The new determination does not seem to justify any corrections of the old values. In the length measurements the value 0.490 m was used throughout.

The base line value on the velocity gave the following first order sides:

1st order triangle side	Geodimeter	Coordinates	Nr obs.
Masmo–Lovö	11032.01 ± 0.03 m	2.06 ± 0.05	2
Prästgrundet–Storjungfrun	20203.60 ± 0.04 »	3.60 ± 0.10	7
Ounistunturi–Sautusvaara	30921.52 ± 0.07 »	0.76 ± ?[1]	17
Kullen–Knösen	23506.33 ± 0.04 »	6.25 ± 0.07	17
Kindlahöjden–Elfhöjden, May	28071.76 ± 0.03 »	1.88 ± 0.10	28
» » , Oct.	.79 ± 0.05 »		32
» » , Nov.	.74 ± 0.05 »		1

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[1] 300 km to base line.

On an average, there was four nights of measurements for each distance. The first two coordinate values are slightly altered as compared with 49, p. 142. This is due to renewed triangle measurements. The limits of the geodimeter values allow for an error in temperature of 0°.5 C.

The distance Kindl.–Elfh. has been choosen with special regard to yield disadvantageous weather conditions. The first and last measurements in the table were carried out by Mr. R. SCHÖLDSTRÖM. He used a new model of the geodimeter. The last sole observation includes four readings (p. 4), which gave a rough value of the limits. At this occasion the weather was as in full winter and the frost nebels made every further measurements impossible. The figures from the side $K-E$ indicate that generally the atmospheric conditions are known with satisfactory accuracy. They also confirm that the good inner agreement among the velocity values is not at random.

A new geodimeter

The weight of the apparatus described above is 250 pounds. The power consumption requires a 450 watt petrol motor-generator. A new model, just completed by AGA, has a weight of 130 pounds and a power consumption of 175 watt. It is furnished with two crystals. They have a difference in frequency of 1 %. By this arrangement it is possible to determine N in eq. (8). The two frequencies are always fixed. The adjustment takes place by electrically varying the time t_1 in eq. (7). The required delay line permits a displacement of the zero-points equal to the distance between two adjacent zero-points. The delay line can be checked by a built in variable light-path. The sensitivity of the new model seems to be increased. Short tests were made in Nov. 1950 and June 1951 (foot and top line of the table).

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